

3D τ_{RR} -minimization in AdS_4 gauged supergravity

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ABSTRACT: In this paper we propose the identification in AdS_4 $\mathcal{N} = 2$ gauged supergravity of the coefficient τ_{RR} of 3D $\mathcal{N} = 2$ SCFTs. We constraint the structure of this function in supergravity by combining the results from unitarity, holography and localization. We show that our conjectured function is minimized by the exact R-charge, corresponding to a gravitational attractor for the scalars in the special geometry. We identify this mechanism with the supergravity dual of the τ_{RR} -minimization. We check this proposal in the ABJM model, comparing with the expectations from localization and the AdS/CFT duality. We comment also on some possible relations with the black hole microstate counting, recently obtained from the application of localization techniques.

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1 Overview

The comprehension of the mathematical structure of the renormalization group (RG) flow is one of the main goals of modern theoretical physics. A general expectation is that the RG flow is irreversible and the degrees of freedom reduce when an ultraviolet (UV) theory flows to the infrared (IR). There have been many proposals for quantifying this idea. In even dimensions exact results have been obtained with the aid of global anomalies. The existence of a monotonically decreasing function interpolating between the UV and the IR fixed points of an RG flow has been proven in 2D [1] and 4D [2]. In 2D this function coincides, at the UV and at the IR fixed points, with the Weyl anomaly, the central charge c . In 4D the role is played by the coefficient of the Euler density, the central charge a . These identifications led to the names of c -theorem in 2D and a -theorem in 4D.

In supersymmetric field theories the central charge a has been computed non-perturbatively from the current correlators of the three point functions [3], and it is

$$a = \frac{3}{32} (\text{Tr} R^3 - \text{Tr} R) \quad (1.1)$$

where R represents the charge associated to the $U(1)_R$ symmetry. In the superconformal case R is related to the scaling dimension Δ . This relation is a consequence of the fact that the R -current is the lowest component of the stress tensor supermultiplet. This current is not in general the UV R -current J_0 , because during an RG

flow J_0 mixes with the other global abelian flavor symmetries F_i . At the fixed point the exact R-current becomes

$$J_R = J_0 + \delta_i F^i \quad (1.2)$$

The current J_R corresponds to the choice of the δ_i coefficients ¹ maximizing the central charge a [4]. This principle has been named a -maximization.

Superconformal field theories received a large attention because of their role in the holographic correspondence. Indeed the correlators of the strongly coupled d -dimensional CFT can be computed at the boundary of the $d + 1$ dimensional dual theory. The correspondence allowed to identify the central charge with the superpotential of the dual $\mathcal{N} = 2$ AdS₅ gauged supergravity theory [5]. The key role in this identification was played by the Chern-Simons coefficients of the gauged 5d theory. The holographic dictionary translated them into the anomalous coefficients of the three point functions of the global currents on the field theory side. This observation, and the identification of the field theory R-charges with the scalars in the very special geometry of $\mathcal{N} = 2$ AdS₅ gauged supergravity, allowed to reformulate a -maximization in this language.

A similar situation has been worked out in the last years in 3D. Here the situation is more complicate, because of the absence of global anomalies. More involved techniques are necessary to test the ideas about the irreversibility of the RG flow. Also the counterpart of a -maximization is not immediately obvious.

This last problem was first solved in [6], where it has been shown that the coefficient C_T of the two point functions for the stress energy can determine the exact R-current. This quantity is related by supersymmetry to the coefficient τ_{RR} of the two point function of the R-current. It has been shown that τ_{RR} is minimized by the exact R-current. This result holds also in 4D as a corollary of a -maximization. Despite the simplicity of this relation the analysis of this quantity is difficult because it cannot be extracted from non perturbative analysis, making the method rather inefficient.

The breakthrough has then been achieved by the application of localization techniques. It has been shown indeed that the free energy computed on S^3 , F_{S^3} , is extremized by the exact R-current [7]. By squashing this manifold it has been proved that the free energy is also maximized by the exact R-charge, essentially for the same reason for which τ_{RR} is minimized [8]. At the fixed point it has also been shown that τ_{RR} and the free energy are proportional [9], even though the general functional relation in terms of the mixing with the R-charge is not known.

¹Observe that the coefficient of the current J_0 depends of the normalization of the R-current. Here we discuss the canonical normalization. In general the difference between an R-current and a non-R-current is the fact that the superpotential is charged under the former but not under the latter.

The parallelism with the 4D case naturally turned the interest to the supergravity side. In this case the calculation of F_{S^3} requires a holographic calculation in the AdS_4 space with Euclidean signature. This analysis has been performed in [10] and the large N behavior of the ABJM model has been reproduced. More recently a non perturbative analysis of the dual mechanism of localization in holography has been performed in [11].

The relation between the τ_{RR} -function and the AdS_4 case with Lorentzian signature has nevertheless been overlooked. It has been shown that one can obtain the τ_{RR} -function in AdS_4 gauged supergravity by consistent truncation of M -theory on SE_7 manifolds [12], but the approach requires the knowledge of the full 10D geometry.

In this paper we initiate the study of the τ_{RR} -function in $\mathcal{N} = 2$ AdS_4 gauged supergravity, in presence of a generic (dyonic) gauging. We use the holographic dictionary to associate the relevant field theory quantities, the charges and the currents, to the scalar fields and the gauge coupling in the supergravity description. This dictionary allows us to propose the supergravity dual τ_{RR} -function. We show that its minimization corresponds to the attractor mechanism for the scalars of the special geometry, while the minimization corresponds to requirement of positivity for the metric on the special manifold. By including the hypermultiplets in the analysis we further constraint the set of charges involved in the minimization. They provide a counterpart of the constraints imposed by the superpotential interactions in the dual field theory.

The paper is organized as follows. In section 2 we review the field theory aspects of τ_{RR} maximization that will be useful for our discussion. We comment on the fixed point relation between the τ_{RR} -function and F_{S^3} computed at large N . In section 3 we review some basics aspects of gauged supergravity that will be relevant in our derivation of the holographic dual of τ_{RR} -minimization. Section 4 contains the main result of the paper. We identify the holographic dual τ_{RR} -function and show its minimization from gauged supergravity. This is done by identifying the R-current from the combination of vector fields that appear in the gravitino variation. This corresponds to a combination of the constrained scalars in the special geometry with the massless gauge fields. The exact R-current is the combination corresponding to the graviphoton of the $\mathcal{N} = 2$ vacuum. By combining the constraint from the special geometry, the holographic dictionary and the results from localization we identify the supergravity dual τ_{RR} . It corresponds to the quartic power of the superpotential appearing in the fermionic variations. In section 5 we discuss the ABJM model to show the whole procedure at work. In section 6 we compare the off-shell behavior of the τ_{RR} -function of the ABJM model expected from AdS/CFT duality, and discuss the possibility of a general relation at large N between the function τ_{RR} and the free energy, in terms of a generic assignment of R-charges. In section 7 we emphasize the possible connection with the recent counting of microstates of 2D AdS_2 black holes

and an 1D R-charge extremization principle. In section 8 we conclude. In appendix A we give further details of AdS₄ gauged supergravity.

2 τ_{RR} -minimization

In this section we review some aspects of τ_{RR} -minimization as discussed in [6]. The τ_{RR} -minimization is a method to obtain the exact R-charge of a superconformal field theory among all the possible choices allowed by the superconformal algebra. The proof of this statement is based on the analysis of the correlation functions of the two point global currents in superconformal field theories:

$$\langle j_i^\mu(x) j_j^\nu(y) \rangle = \frac{\tau_{ij}}{2\pi^3} (\partial^2 \delta^{\mu\nu} - \partial^{\mu\nu}) \frac{1}{(x-y)^2} \quad (2.1)$$

where, because of unitarity, the matrix τ_{ij} has positive eigenvalues. In superconformal field theories one of these global currents corresponds to the lowest component of the supermultiplet having the stress energy tensor as highest component. This is commonly referred as the R-current.

This current is in general a combination of the UV R-current R_0 and other flavor current F_i . Defining a trial R-current

$$R_t = R_0 + \delta_i F_i \quad (2.2)$$

The exact R-current corresponds to a specific assignment of the coefficients δ_i . Some combinations in (2.2) are usually excluded by the structure of the interactions but this is not enough, in general, to fix the coefficients δ_i . A closer look at the correlation functions imposes the necessary constraints. The coefficient $\tau_{R_t R_t}$ is

$$\tau_{R_t R_t} = \tau_{R_0 R_0} + 2 \sum \delta_i \tau_{R_0 i} + \sum_{i,j} \delta_i \delta_j \tau_{ij} \quad (2.3)$$

This function is minimized by the choice of coefficients δ_i that correspond to the exact R-current. This has been proved by studying the first and the second derivatives of τ_{RR} in the δ_i space. They correspond to $\tau_{R_0 i}$ and τ_{ij} . The first is set to zero by supersymmetry and imposes an extremization condition. The minimization condition is imposed by the unitarity of the matrix τ_{ij} .

Despite the simplicity of this result the τ_{RR} extremization did not become a popular method to extract the R-charge, because of the absence of anomalies in 3D. The general non-perturbative structure of the τ_{RR} -function is not known and the function can be used to compute the exact R-charge only at weak coupling in the perturbative expansion. A more tractable object is the free energy F_{S^3} , computed by localization of the path integral on S^3 [7, 13, 14]. This led to a non perturbative exact result and it was shown that the F_{S^3} is maximized by the exact R-charge [8].

Moreover the non perturbative nature of this function allowed the conjecture of a 3D F -theorem [15]. On the contrary counterexamples to a τ_{RR} theorem have been found in [16].

When considering large N supersymmetric gauge theories it has been shown that at the fixed point the large N free energy and τ_{RR} are related by a simple relation [9]

$$F_{S^3}^{max} = \frac{\pi^2}{4} \tau_{RR}^{min} \quad (2.4)$$

In the rest of the paper we look for the τ_{RR} -function and its minimization principle from the point of view of $\mathcal{N} = 2$ AdS₄ gauged supergravity.

3 AdS₄ gauged supergravity

In this section we review some general aspects of AdS₄ gauged supergravity. This allows us to fix the notations necessary for the rest of the analysis. We provide further details in Appendix A.

At the anti de Sitter vacuum fermionic fields are set to zero, thus the relevant dynamics is given by the bosonic part of the action. In $\mathcal{N} = 2$ Supergravity, coupled to n_V vector multiplets and n_H hypermultiplets, it can be written as

$$S = \int d^4x \left(-\frac{R}{2} + \mathcal{I}_{\Lambda\Sigma} F_{\mu\nu}^\Lambda F^{\Sigma\mu\nu} + \frac{1}{2\sqrt{-g}} \mathcal{R}_{\Lambda\Sigma} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^\Lambda F_{\rho\sigma}^\Sigma + \right. \\ \left. + g_{i\bar{j}} \partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}} + h_{uv} \nabla_\mu q^u \nabla^\mu q^v - V_g(z, \bar{z}, q) \right) \quad (3.1)$$

Our study will deal with abelian gaugings, in particular where only the scalars of the quaternionic manifold are charged under the gauge fields, while the scalars of the vector multiplet remain neutral. The latter corresponds to the request that only isometries of the Quaternionic manifolds are gauged, namely that the potential is of the form

$$V_g(z, \bar{z}, q) = 4h_{uv} \langle k^u(q), \mathcal{V}(z, \bar{z}) \rangle \langle k^u(q), \bar{\mathcal{V}}(z, \bar{z}) \rangle - 3W\bar{W} + g^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} \quad (3.2)$$

We are introducing here a complex function built from the product of the moment maps with the symplectic sections as²

$$W(z, \bar{z}, q) = \langle \mathcal{P}^x(q), \mathcal{V}(z, \bar{z}) \rangle \equiv e^{K/2} (\mathcal{P}_\Lambda^x X^\Lambda - \mathcal{P}^{x\Lambda} F_\Lambda) \quad (3.4)$$

²We will generically consider both electric and magnetic gauging, thus we indicate

$$\mathcal{P}^x = \mathcal{P}_\lambda^x (\Theta^{\Lambda\lambda}, \Theta_\Lambda^\lambda), \quad (3.3)$$

for a generic choice of embedding tensor Θ [17]. Analogously the Killing vectors will be given in a symplectic vector $k^u = (k^{u\Lambda}(q), k_\Lambda^u(q))$.

that reduces to the domain walls superpotential in the case of $U(1)$ R-symmetry gauging (Fayet-Iliopoulos). For a complete account of definitions of special geometry and gauged supergravity we refer to the appendix.

A configuration with zero fermions and zero gauge fields is supersymmetric if the corresponding supersymmetry variations of the fermions vanish. They involve the scalar fields and are explicitly given by

$$\begin{aligned}\delta\psi_\mu^A &= D_\mu\epsilon^A - \frac{1}{2}(\sigma^x)_B^A\gamma_\mu W\epsilon^B + \dots \\ \delta\lambda^{iA} &= ig^{i\bar{j}}(\sigma^x)_B^A D_{\bar{j}}\bar{W}\epsilon^B + \dots \\ \delta\zeta^\alpha &= \mathcal{U}_{u\alpha}^A\langle k^u, \bar{\mathcal{V}}\rangle\epsilon_A + \dots\end{aligned}\tag{3.5}$$

where the dots indicate terms which are identically zero at the vacuum. In all the cases we consider we are always able to use $SU(2)$ symmetry to rotate the moment maps \mathcal{P}^x in the direction \mathcal{P}^3 , with $\mathcal{P}^1 = \mathcal{P}^2 = 0$ (in particular this is how definition (3.4) has to be read). Notice that there are cases where this is not possible [18, 19]. This requires modifications in the analysis and we will leave this point to further investigations.

The conditions for the supersymmetric vacuum are then

$$\partial_i|W| \big|_{\{q^{u*}, z^{i*}, \bar{z}^{i*}\}} = 0, \quad \langle k^u(q^*), e^{-K/2}\mathcal{V}(z^*) \rangle = 0\tag{3.6}$$

where the extremization is done only over the scalars z^i of the Special Kähler manifold, and $\{q^{u*}, z^{i*}, \bar{z}^{i*}\}$ indicate the value of the scalar fields at the minimum. The other condition depends on how many Killing vectors are identically zero at the vacuum. In particular, the non zero Killing vectors corresponds to a number n_c of algebraic, holomorphic constraints on the fields z^i , related to the Higgsing of n_c abelian vector fields at the vacuum [20]. Moreover, because of special geometry, if the supersymmetric vacuum is given by the scalar configuration (q^*, z^*, \bar{z}^*) , the AdS_4 radius, and thus the cosmological constant at the minimum of the potential can be expressed as

$$-\frac{\Lambda}{3} = \frac{1}{\ell_{AdS}^2} = -\frac{1}{2}\mathcal{P}^{xT}(q^*)\mathcal{M}(z^*, \bar{z}^*)\mathcal{P}^x(q^*)\tag{3.7}$$

by the scalar dependent $Sp(2n_V + 2)$ matrix

$$\mathcal{M}(z^i, \bar{z}^{\bar{i}}) = \begin{pmatrix} \mathcal{I} + \mathcal{R}\mathcal{I}^{-1}\mathcal{R} & -\mathcal{R}\mathcal{I}^{-1} \\ -\mathcal{I}^{-1}\mathcal{R} & \mathcal{I}^{-1} \end{pmatrix}\tag{3.8}$$

where $\mathcal{I} \equiv \mathcal{I}_{\Lambda\Sigma}$ and $\mathcal{R} \equiv \mathcal{R}_{\Lambda\Sigma}$ are the matrices of the scalar coupling to the gauge fields as taken from the Lagrangian in (3.1). It is important to notice that at the extremum (3.6), the inverse of AdS length square is given by the square root of

the quartic symplectic invariant $\mathcal{I}_4(\mathcal{G})$ [21, 22] valued at the “charges” given by the moment maps evaluated at the vacuum as

$$\mathcal{G} \equiv \mathcal{P}^x(q^*) . \quad (3.9)$$

This follows immediately from the study of black hole horizons attractors [23–25], where exactly the same extremization occurs with respect to the scalars of the vector multiplets z^i . In that case the charges \mathcal{G} are the black hole charges and the quartic invariant gives the value of the black hole entropy, or better, the volume of the S^2 at the horizon (black hole area).

4 τ_{RR} in $\mathcal{N} = 2$ gauged supergravity

In this section we study the τ_{RR} -minimization from gauged supergravity. We start our analysis with the identification of the conserved currents and of the charges that determine their mixing. Observe that there can be other broken global currents, that we ignore in this first part of the discussion. In other words we first restrict the analysis to the case in which the second equation in (3.6) is solved by setting k^u to zero. This corresponds of restricting our attention to the sector of the conserved currents that mix with the R -charge, with constant moment maps.

The photons appearing in the supersymmetric variations of the gravitino and of the gaugino are identified with the R and the conserved global currents of the dual field theory. In general the photons that appear in the supersymmetric variations of the fermions are combined with the superpotential. In the case of the R -current the combination is proportional to the superpotential while in the case of the gaugino there is also a derivative involved. This distinguishes the R current from the other conserved global currents. In the dual field theory this translates into the fact that the supercharges are charged under the R -current while they are uncharged under the non- R globally conserved currents. The coefficients of the mixing can be read from the variation of the gravitino. In general they are proportional to the symplectic sections \mathcal{V} defined in (3.4). In formulas by referring to a symplectic vector of charges as s we have $s = t\mathcal{V}$.

The coefficient of the normalization ³ is imposed by normalizing the charge of the gravitino to 1. This choice corresponds to $t = 1/W$ as can be read from the gravitino variation. Our R charges become

$$s = \frac{\mathcal{V}}{W} \quad (4.1)$$

³One can choose also other normalizations, it is just important to consider this difference when mapping the normalization of the charges described in supergravity with the ones on the field theory side.

The flavor currents are obtained by acting with derivatives on \mathcal{V} . The combinations of the charges that determines the exact R -current is determined by (3.6).

At this point we can try to identify the τ_{RR} -function in AdS_4 gauged supergravity. The coefficient of the flavor two point function in the AdS/CFT correspondence is dual to the inverse square Yang Mills coupling in the holographic correspondence. This observation is the starting point to identify the τ_{RR} -function. Here we adapt the discussion of [12] to the symplectic invariant formalism. In this case we can use the matrix \mathcal{M} in formula (3.8) and our starting point becomes the formula

$$\tau_{RR} = \mathcal{T} s^T \mathcal{M} \bar{s} \quad (4.2)$$

We observe that this is a real function, depending on a combination of complex R -charges. This notion requires some interpretations. In a magnetic gauging we can restrict to the simpler formula $\tau_{RR} \propto \mathcal{I}_{\Lambda\Sigma} s^\Lambda \bar{s}^\Sigma$. In this case one can set the imaginary parts of the sections to zero and treat the R -charges as real. An analogous discussion holds for an electric gauging. The situation is more subtle in a dyonic gauging. In such a case one should apply a symplectic rotation to identify the correct real combinations leading to the R -charges. As discussed in section 2 some dyonic gaugings are more subtle [18, 19] and deserve further investigations.

We insert the R -charges (4.1) in (4.2) and simplify the expression by using the constraints of the special geometry. In this way we obtain

$$\tau_{RR} = \mathcal{T} \mathcal{M} s \bar{s} = \frac{\mathcal{T}}{|W|^2} \quad (4.3)$$

The extremization condition corresponds to the requirement of an $\mathcal{N} = 2$ AdS vacuum (3.6). The signs of the second derivatives are imposed from the constraints of the special geometry. The sign is determined by the positivity of the scalar metric in (A.2). Here we obtain

$$\partial_i \partial_{\bar{j}} |W|^{-2} \propto -g_{i\bar{j}} |W|^{-2}, \quad (4.4)$$

This relation does not seem to be correct, because the second derivative of τ_{RR} should have the opposite sign. This leads to a maximization of τ_{RR} and not to a minimization as expected.

Here we discuss a possible resolution of this problem, similarly to the discussion of [12], where τ_{RR} was computed from the AdS/CFT correspondence. Here we observe that the function τ_{RR} at the fixed point is reproduced by the relation

$$\tau_{RR} = \mathcal{T} \frac{|W|^{p-2}}{\mathcal{I}_4^{p/4}} \quad (4.5)$$

This extremal value of this function and its first derivative are independent from the value of p . Notice that the Hessian at the extremum $\partial_i |W| = 0$ can be obtained by using (A.8) and it is given by

$$\partial_{\bar{j}} \partial_i \tau_{RR} = g_{i\bar{j}} (p-2) \tau_{RR} . \quad (4.6)$$

Different values of p are consistent with the extremization of the conjectured τ_{RR} -function but they can lead to a maximization instead of a minimization problems. We can try to speculate on the origin of the different values of p . In the holographic dictionary the coefficient τ_{RR} , in generic dimensions d , is a function of the AdS length scale, ℓ_{AdS}^{d-3} . For the case $d = 3$ the dependence of τ_{RR} from ℓ_{AdS} drops out. When we study the behavior of the function τ_{RR} we vary the scalars in the vector multiplets, while keeping the moment maps constant, i.e. the hyperscalars fixed at their supersymmetric vacuum. In view of this observation the relation (3.7) inserts back the AdS scale in the problem, introducing another dynamical object in the minimization problem. It does not modify the dictionary explained above, but it can modify the derivatives and the off-shell behavior of τ_{RR} .

Another source of mismatch resides in the identification of the R -charges. If we perform a symplectic rotation and reduce to an electric gauging we can identify the graviphoton with the formula $e^{\mathcal{K}/2} X^\Lambda \mathcal{I}_{\Lambda\Sigma} A_\mu^\Sigma$. The R charges in this case are given by the relation $s^\Lambda = X^\Lambda / (X^\Lambda P_\Lambda)$. The structure of the graviphoton is read from the relation above by evaluating $\mathcal{I}_{\Lambda\Sigma}$ at the fixed point. At the fixed point this relation gives the correct mixing of the charges. Out of the fixed point nevertheless the matrix $\mathcal{I}_{\Lambda\Sigma}$ is a function of the sections. This can be another source of problems in the identification of τ_{RR} in (4.3). In the rest of the analysis we will fix $\mathcal{I}_{\Lambda\Sigma}$ to its constant value when identifying the charges of the photons. It would be interesting to come back to this problem.

In the rest of the analysis we fix the coefficient of p above to match the off shell relation that one can guess from the AdS/CFT analysis of [12]. Here we propose that the correct power is obtained for $p = 6$. This is a conjectural choice and we will be more concrete on this relation in section 6. The τ_{RR} -function becomes

$$\tau_{RR} = \mathcal{T} \frac{|W|^4}{\mathcal{I}_4^{3/2}} \quad (4.7)$$

Observe that this discussion is valid in absence of trivial magnetic fluxes. We will comment on their role in section 7. Moreover, eq. (4.7) suggests that the functional dependence of τ_{RR} on the charges is $\tau_{RR} \propto (s^T \mathcal{M} \bar{s})^{-2}$.

In this way we have obtained the supergravity dual of the τ_{RR} -minimization principle discussed in [12]. We can also fix the proportionality constant \mathcal{T} from the relation (2.4) and from the relation $|W^{min}|^4 = \mathcal{I}_4$. We obtain

$$\tau_{RR}^{min} = \mathcal{T} \frac{|W^{min}|^4}{\mathcal{I}_4^{3/2}} = \frac{4}{\pi^2} F_{S^3}^{max} \quad \rightarrow \quad \mathcal{T} = \frac{4}{\pi^2} F_{S^3}^{max} \sqrt{\mathcal{I}_4} \quad (4.8)$$

By imposing this normalization our candidate supergravity dual τ_{RR} -function can be written as

$$\tau_{RR} = \frac{4 F_{S^3}^{max}}{\pi^2} \frac{|W|^4}{\mathcal{I}_4} \quad (4.9)$$

We conclude this section by including in the discussion the effect of broken global symmetries. In 3D there are not constraints coming from the global anomalies but there are superpotential couplings, that break some of the global symmetries that can mix with the R-charge. One may wonder if the effect of these couplings can be captured by the τ_{RR} function. This idea is borrowed from the 4D case.

In 4D the effects of the broken symmetries are captured in a -maximization by including some Lagrange multipliers in the problem. The multipliers impose the superpotential (and the anomaly) constraints and they are associated to the coupling constants. It has been shown [26, 27] that this procedure matches the perturbative results in field theory, i.e. one can expand the exact R-charges in terms of the multipliers and match with the perturbative expansion. In 3D a similar proposal for τ_{RR} is missing, but it has been shown that the multipliers can be considered in the extremization problem of F_{S^3} [28], where a two loop matching was observed. This allows us to think that a similar mechanism can be proposed for τ_{RR} and motivates the search of the supergravity dual of the Lagrange multipliers.

First we have to translate the effect of the broken global symmetries. They are related to the presence of massive gauge bosons on the holographic dual side. This effect can be captured in presence of hypermultiplets. So far we only considered the hypermultiplets fixed at their supersymmetric vacuum and we used them only to determine the correct moment maps necessary to identify the mixing of the currents. Here we turn on some of these fields, considering their role in the extremization of τ_{RR} .

The effects of the hypermultiplets is obtained by a closer look at the supersymmetric variations. In the discussion above we have considered only the solution $k^u = 0$ to (3.6). Now we expand around these solutions, and consider some $k^u \neq 0$, at linear order in the hypermultiplets. These solutions split \mathcal{M}_H into two parts, \mathcal{M}_{H_1} and \mathcal{M}_{H_2} . For the \mathcal{M}_{H_1} submanifold the situation corresponds to the one described above: k^u are vanishing and they do not provide further constraints on \mathcal{V} . Here we concentrate on the submanifold \mathcal{M}_{H_2} . In this case the n_{H_2} non vanishing k^u signal the spontaneous breaking of the gauge group. There are $n_{H_2} \leq n_V$ massive gauge bosons, and they acquire the mass by an Higgs mechanism, eating n_{H_2} scalars in \mathcal{M}_{H_2} , leaving only $3n_{H_2}$ scalars on \mathcal{M}_{H_2} . The moment maps \mathcal{P}^x become functions of the uneaten hyperscalars. Here, as discussed above, we can use still an $SU(2)$ transformation when expanding around the vacuum, and consider only one non vanishing component, \mathcal{P}^3 , depending on n_{H_2} coordinates on \mathcal{M}_{H_2} . This is valid if the Killing potentials are expanded around the supersymmetric vacuum at linear order in the hyperscalars [5]. At the same time the solution of the hyperino variation for the non vanishing components of k^u impose n_{H_2} conditions on the scalars on \mathcal{M}_V . They are exactly the n_{H_2} constraints imposed by the n_{H_2} hyperscalars in \mathcal{P}^3 .

This discussion coincides with the one of AdS_5 gauged supergravity done in

[5]. This led to identify the multipliers of the field theory description with the hyperscalars in the moment map \mathcal{P}^3 enforcing the constraints on \mathcal{V} . Here we see the same mechanism at work on the gravity side. The presence of the multipliers allows the study of R-symmetric RG flows, along the lines of [29] (see also [30] for another discussion on the role of the multipliers in the AdS_5 case).

5 The τ_{RR} function of the ABJM model

In this section we apply our formalism to the calculation of the holographic τ_{RR} -function for the ABJM model.

In general the simplest example that one can consider consists of $\mathcal{N} = 2$ gauged supergravity with a graviton multiplet and n_V vector multiplets. Here we choose the case with $n_V = 3$ and focus on a solution discussed in [31]. This theory corresponds to a consistent truncation of S^7 , and it accounts for a deformation of the ABJM model, along the Cartan $U(1)^4$ of the $SO(8)_R$ symmetry group. The model admits a formulation in terms of a prepotential

$$\mathcal{F} = -2\sqrt{-X^0 X^1 X^2 X^3} \quad (5.1)$$

The symplectic vector \mathcal{V} and the Kähler potential \mathcal{K} are

$$\mathcal{V} = e^{\mathcal{K}/2}(1, t_2 t_3, t_1 t_3, t_1 t_2, -it_1 t_2 t_3, -it_1, -it_2, -it_3), \quad e^{-\mathcal{K}} = 8\text{Re}(t_1)\text{Re}(t_2)\text{Re}(t_3) \quad (5.2)$$

We consider a purely electric gauging, with charges $g_i = \frac{1}{2}$ ($i = 1, \dots, 4$). The moment maps are constant functions of these charges. They can be formulated in terms of \mathcal{P}^3 as

$$\mathcal{P}^3 = \frac{1}{2}(0, 0, 0, 0; 1, 1, 1, 1) \quad (5.3)$$

The τ_{RR} -function in this case is

$$\tau_{RR} = \frac{F_{S^3}^{max}}{64\pi^2} \frac{|1 + t_1 t_2 + t_1 t_3 + t_2 t_3|^4}{(\text{Re}(t_1) \text{Re}(t_2) \text{Re}(t_3))^2} \quad (5.4)$$

Where $F_{S^3}^{max}$ corresponds to the maximal value of the F_{S^3} , used here as an input, obtained from localization. At the extremal point the scalars t_i are fixed to the $\mathcal{N} = 2$ supersymmetric vacuum, $t_i = 1$. This is the supersymmetric attractor mechanism and it corresponds to the condition (3.6). In the field theory interpretation this corresponds to the extremization condition on the R-charges (4.1). The τ_{RR} function evaluated at the minimum corresponds to the expected result (2.4).

We can also compute the R-charges. In this case we are in presence of a magnetic gauging and the graviphoton can be written in terms of the sections as $e^{\mathcal{K}/2} X^\Lambda F_\Lambda^{\mu\nu}$. As discussed above here we describe the charges for the field strength $\mathcal{I}_{\Lambda\Sigma} F^{\Sigma\mu\nu} \equiv F_\Lambda^{\mu\nu}$. In this sense we treat $\mathcal{I}_{\Lambda\Sigma}$ as a constant matrix. Moreover the sections can be treated

as real. We choose the R-charges in this case as $s^\Lambda = e^{\mathcal{K}/2} X^\Lambda / 2\mathcal{W}$. We can also turn off the imaginary parts the coordinates $t_i = b_i + iv_i$ and parameterize the R-charges as

$$s^1 = \frac{2}{B} \quad s^2 = \frac{2b_2b_3}{B} \quad s^3 = \frac{2b_1b_3}{B} \quad s^4 = \frac{2b_1b_2}{B} \quad (5.5)$$

where we defined $B = 1 + \sum_{i < j} b_i b_j$. At the fixed point the scalars are $b_i = 1$ and the R-charges s^Λ are all equal to $\frac{1}{2}$. At $k = 1$ the normalization τ_{RR}^{min} can be extracted from [15]. It is

$$\tau_{RR}^{min} = \frac{4F_{S^3}^{max}}{\pi^2} = \frac{4\sqrt{2}}{3\pi} N^{3/2} \quad (5.6)$$

Observe that in this case we can describe the function τ_{RR} before the extremization as a function of the R-charges s^Λ . We have

$$\tau_{RR} = \frac{\sqrt{2}}{3\pi} N^{3/2} \frac{1}{16 s^1 s^2 s^3 s^4} \quad (5.7)$$

It is interesting to compare this result with the expectations from localization. Indeed in this case we can associate the R-charges to the one of the ABJM model. This model can be associated to a consistent truncation of a deformed ABJM model. On the field theory side the ABJM model corresponds to a 3D quiver gauge theory with $U(N)_k \times U(N)_{-k}$ gauge groups, where k is an integer Chern-Simons (CS) level. There are two pairs of bifundamental fields a_i and b_j with superpotential

$$W = a_1 b_1 a_2 b_2 - a_1 b_2 a_2 b_1 \quad (5.8)$$

The theory has $\mathcal{N} = 6$ supersymmetry enhanced to $\mathcal{N} = 8$ for $k = 1, 2$. The model that we considered in this section, corresponding to set the FI parameters of the gauging $g_i = 1/2$, is a consistent truncation of the ABJM model preserving the full Cartan $U(1)^4$ symmetry. The situation with generic g_i corresponds to the topological twist discussed in [32]. We will come back to this case in section 7.

The free energy can be parameterized on the field theory side with the R-charges of the four fields a_1 , a_2 , b_1 and b_2 . The general parameterization respecting the $U(1)^4$ Cartan symmetry is

$$\begin{aligned} \Delta_{a_1} &= \delta_0 + \delta_1 + \delta_2 + \delta_3, & \Delta_{a_2} &= \delta_0 + \delta_1 - \delta_2 - \delta_3 \\ \Delta_{b_1} &= \delta_0 - \delta_1 + \delta_2 - \delta_3, & \Delta_{b_2} &= \delta_0 - \delta_1 - \delta_2 + \delta_3 \end{aligned} \quad (5.9)$$

The free energy at large N is

$$F_{S^3} = \frac{\sqrt{2}\pi}{3} N^{3/2} \sqrt{\Delta_1 \Delta_2 \Delta_3 \Delta_4} \quad (5.10)$$

The maximization of the free energy fixes $\Delta_{a_i} = \Delta_{b_i} = \frac{1}{2}$. The exact R-current is obtained by the combination ⁴

$$R_{ex} = \delta_0 J_0 + \delta_i F_i \quad (5.11)$$

⁴Here we refer to the canonical normalization of the R symmetry $R_0 = (d-2)/2J_0$, this explains the difference in the normalization discussed in (2.2).

where F_i represent the three $U(1)$ currents that can mix with the R-charge. In this case, at the fixed point we have $\delta_i=0$. The canonically normalized exact R-current corresponds to $J_0/2$. This is an expected result, because in this case two of the global $U(1)$ s are actually in the Cartan of the global $SU(2)^2$ symmetry group and the other $U(1)$ is a baryonic symmetry.

We want to compare this result in terms of the supergravity result obtained in (5.7) for τ_{RR} . The graviphoton in this case corresponds to the combination $s^\Lambda A_\Lambda^\mu$, where the s^Λ are the charges parameterized in (5.5). The relation between the s^Λ and the δ_i variables is

$$\begin{aligned}\delta_0 &= \frac{s^0 + s^1 + s^2 + s^3}{2}, & \delta_1 &= \frac{s^0 + s^1 - s^2 - s^3}{2} \\ \delta_2 &= \frac{s^0 - s^1 + s^2 - s^3}{2}, & \delta_3 &= \frac{s^0 - s^1 - s^2 + s^3}{2}\end{aligned}\tag{5.12}$$

supplemented by the constraint $s^\Lambda P_\Lambda = 1$, that here becomes $\sum s^\Lambda = 2$. At the vacuum this reproduces the expected relation $\delta_0 = 1$ and $\delta_i = 0$.

To conclude this section we want to compare the functional behavior of the function τ_{RR} and the function $F_{S^3}^2$, in terms of the parameterization found in (5.12). We obtain the relation $F_{S^3}^2(s^\Lambda) \propto \tau_{RR}^{-1}(s^\Lambda)$. This relation is the one expected from the AdS/CFT correspondence, as we will comment in section 6. This is consistency check of our conjecture on the structure of τ_{RR} .

6 Relation with the large N F_{S^3} and $Vol(SE_7)$

In this section we comment on the relation between our results and the predictions from localization. Here we conjectured the structure of τ_{RR} compatible with a functional relation between τ_{RR} and F_{S^3} in terms of the R-charges. Such a relation is indeed expected from the results of [12], where the relation between τ_{RR} and the volume form $Vol(Y)$, appearing when studying M-theory compactified on a SE_7 manifold Y , was discussed.

In general F_{S^3} and τ_{RR} are different functions in terms of their dependence on a generic assignment of the R-charges. Nevertheless, as we observed in the case of the ABJM theory, our conjectured definition of τ_{RR} leads to the functional relation $\tau_{RR} \propto F_{S^3}^{-2}$, once the R-charges on the two sides of the duality are identified. This corresponds to the choice $p = 6$ in (4.5).

Let us briefly review the results of [12]. In AdS/CFT the volume $Vol(Y)$ is parameterized in terms of the Reeb vector \mathbf{b} . This vector is a Killing vector corresponding to one of the $U(1)$ isometries of Y . This isometry corresponds to the R-symmetry in the dual field theory. The exact R-charge is obtained by minimizing the volumes in terms of the components of \mathbf{b} [33]. It was observed that τ_{RR} , obtained

from the KK reduction on the volume formula, is proportional, at the fixed point, to the inverse volume. This led to an apparent contradiction, being both the functions⁵ $\tau_{RR}(\mathbf{b})$ and $Vol_{\mathbf{b}}(Y)$ minimized by the exact R-charge. The way out discussed in the paper was to distinguish a functional dependence of $\tau_{RR}(\mathbf{b})$ from $Vol_{\mathbf{b}}(Y)$ and a normalization to respect of the volume at the fixed point, $Vol_{min}(Y)$. In this way it does still make sense to have two different principles of minimization for $\tau_{RR}(\mathbf{b})$ and $Vol_{\mathbf{b}}(Y)$ but an inverse functional dependence from the R-charge parametrized by the components of the vector \mathbf{b} . In this paper we observed a similar mechanism at work in gauged supergravity. The relation between τ_{RR} and the volume is [12]

$$\tau_{RR}(\mathbf{b}) = \frac{4\pi^2}{3\sqrt{6}} \left(\frac{N}{Vol_{min}(Y)} \right)^{3/2} Vol_{\mathbf{b}}(Y) \quad (6.1)$$

On the other hand the general relation between the free energy and the volume $Vol(Y)$ is

$$F_{S^3}(\mathbf{b}) = N^{3/2} \sqrt{\frac{2\pi^6}{27Vol_{\mathbf{b}}(Y)}} \quad (6.2)$$

By combining the two relations (6.2) and (6.1) one obtains

$$\tau_{RR} = \frac{4}{\pi^2} \frac{(F_{S^3}^{max})^3}{F_{S^3}^2} \quad (6.3)$$

where $F_{S^3}^{max}$ is the maximized free energy corresponding to $Vol_{min}(Y)$ in (6.1). This leads to a prediction for the relation between τ_{RR} and the large N free energy F_{S^3} .

In other words here we fixed $p = 6$ in (4.5) to match the predictions on τ_{RR} for the ABJM model. As we discussed above it would be important to have a more direct derivation of this result from gauge supergravity. This may also shed some light on a general expectation, based on the analogy with the 4D case [34]. The expectation is that the volume form can be always associated to a quartic function of the R-charges [35, 36]. Gauged supergravity has already been proposed in [37] to find a similar relation. We hope to come back to these problems in the future.

7 Topological twist and relation with AdS₂ BH entropy

In this section we speculate on some possible relations with a very interesting result, recently appeared in [32]. In this paper the authors counted the microstates of asymptotically AdS black holes, reproducing the Bekenstein-Hawking (BH) entropy from the calculation of an index in the holographic dual gauge theory. The index is a function of the R-charges of the dual superconformal field theory, and it has been

⁵Here we refer to the τ_{RR} -function computed from the AdS/CFT correspondence. For this reason we express the dependence from the components of the Reeb vector \mathbf{b} .

shown that the correct entropy is found once the exact R-charge is imposed on the index. The exact R-charge is obtained by an extremization principle, dual, on the gravity side, to an attractor mechanism. Because of the odd dimensionality the authors proposed a derivation of this extremization in analogy with the maximization of F_{S^3} , observing that the Witten index on S^1 has indeed the desired properties. Having τ_{RR} the same extremization properties of F_{S^3} one may hope to derive an extremization principle also from our results. Here we observe that there is a possibility of deriving such a relation by reducing to $\text{AdS}_2 \times S^2$ the holographic τ_{RR} -function. When compactifying AdS_4 on $\text{AdS}_2 \times S^2$ with magnetic fluxes turned on, the AdS_4 superpotential W_4 reduces to the ratio of the AdS_2 central charge Z_{2D} and the AdS_2 superpotential W_{2D} .

As we observed above, a generic function, proportional to the superpotential $|W|$, is extremized by the exact R-charge. Nevertheless when we consider the presence of magnetic fluxes they can mix with the R-charge and the various function have different extremization properties. What seem reasonable is to study the function at $p = 2$ in (4.5). This function is maximized by the exact R-charge also in the case when the theory is deformed by the fluxes. Moreover, along the gravitational flow, this function becomes [38]

$$\tau_{RR} = \frac{2}{\pi G_4 |W_4|^2} \rightarrow \frac{1}{G_4} \left| \frac{Z_2}{W_2} \right| \quad (7.1)$$

During this reduction the scalars cannot be kept fixed and their mixing provides a different attractor. In the field theory language the addition of the fluxes is equivalent to a topological twist on the flavor symmetries, and the new attractor can be reformulated by a different mixing of the R-current with the fluxes in the dual 1D superconformal quantum mechanics. The AdS_2 attractor equation in this case fixes the correct mixing and should correspond to an R-charge extremization principle on the field theory side. At the vacuum the relation (7.1) becomes

$$\frac{1}{G_4} \left| \frac{Z_2}{W_2} \right| \propto \frac{R_{S^2}^2}{G_4} \propto S_{BH} \quad (7.2)$$

reproducing the BH entropy. It would be interesting to investigate in this direction. For example one can try to reproduce the results of [32] and further study the case of other BPS black holes, as the ones obtained in [39] from consistent truncations to AdS_4 of 10D M -theory. Our analysis suggests that an R-charge extremization principle at work in the dimensional reduction of the associated topologically twisted field theories may be captured by the reduction of (7.1).

8 Conclusive discussion: open problems and further investigations

In this paper we studied the coefficient of the two point function for the R-current of 3D $\mathcal{N} = 2$ SCFTs from $\mathcal{N} = 2$ AdS₄ gauged supergravity. Taking advantage of the constraints of the special geometry we have conjectured the supergravity dual τ_{RR} -function (4.9). We have derived the extremization principle of [12], to obtain the exact mixing of the R-current with the abelian symmetries, in the gravitational setup. It corresponds to an attractor mechanism for the scalars in the vector multiplets. We discussed also the role of the quaternionic manifold, showing that the hypermultiplets can be interpreted as Lagrange multipliers constraining the extremization. The analysis does not require the existence of a prepotential and it applies for different choices of gauging in many setups.

In the derivation we conjectured the behavior of τ_{RR} in order to reproduce the AdS/CFT predictions in the case of the ABJM model. In this way we obtained the relation between the τ_{RR} -function and the free energy F_{S^3} for general R-charges, matching the expectations from the volume computations. More general checks and studies in this direction are necessary. Here we did not consider other truncations that can have interesting consequences in the AdS/CFT correspondence. For example one can consider the dual theories conjectured in [40–42] for the truncation of M^{111} and Q^{111} and compare with the predictions from the volume formula obtained from the geometry. In this case it would be possible to identify the general behavior of τ_{RR} in gauged supergravity in terms of the Reeb vector for a general truncation of SE₇ manifolds along the lines of [43]. Observe that for some of these theories the calculation of the free energy does not reproduce the $N^{3/2}$ scaling behavior [15]. Our analysis in gauged supergravity may produce a different holographic check for these models.

We also discussed a possible relation between our construction and the results of [32]. Motivated by this relation we think that it would be interesting to perform a direct study of the R-charge extremization problem from the 1D perspective. This analysis is similar to the ones performed in [44–46] when flowing to AdS₅ to AdS₃.

In our analysis we have been interested in cases without higher derivatives terms. The inclusion of these contribution should corresponds to models with non vanishing TrR. Another analysis deserving further investigation is the study of the global properties of the hyperscalar manifold. This is necessary for studying flows between supersymmetric solutions. Here, by including the effects of the Lagrange multipliers, we restricted to the possibility of R-symmetric supersymmetric RG flows.

Here conclude with a last observation. The prepotential $\mathcal{F} = C_{IJK}X^IX^JX^K/X^0$ corresponds to the very special Kähler geometry and it is related to the AdS₅ case. Indeed it can be obtained by reducing the $\mathcal{N} = 2$ AdS₅ supergravity. It would

be interesting to understand if this observation has some possible consequences in the relation between the free energy and the central charge obtained in [47], where an interpolation between 4D a -maximization and 3D F -maximization was obtained. Recently another connection between a and F was discussed in [48]. In this case our analysis may get modified by the presence of a dyonic gauging. One may try to connect these result and the special geometry of the AdS_5 and the AdS_4 supergravity.

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A Definitions and useful identities of gauged $\mathcal{N} = 2$ supergravity

A general $N = 2$ theory⁶ can be coupled to n_V vector multiplets $(A_\mu^I, \lambda^{iA}, \lambda_A^{i*}, z^i)$, containing complex scalar fields z^i ($I, i = 1, \dots, n_V$), and n_H hypermultiplets (ζ^α, q^u) , containing real scalars ($\alpha = 1, \dots, 2n_H$, $u = 1, \dots, 4n_H$).

The $4n_H$ real q^u scalars are in fact coordinates of a quaternionic manifold \mathcal{QM} of quaternionic dimension n_H . The choice of gauging considered in this work involves a group of isometries $G \in \mathcal{QM}$. It is defined by a set of moment maps $\mathcal{P}^x(q)$ related to the Killing vectors as [20]

$$-2k_\Lambda^u K_u^x v = \nabla_v \mathcal{P}_\Lambda^x, \quad (\text{A.1})$$

where K^x is the curvature of the $SU(2)$ connection on the quaternionic manifold.

The complex scalars z^i parametrize a special Kähler manifold \mathcal{SM} , whose geometry is completely defined by a Kähler potential $K(z, \bar{z})$, from which the metric of the manifold is derived as

$$g_{i\bar{j}}(z, \bar{z}) = \partial_i \partial_{\bar{j}} K(z, \bar{z}). \quad (\text{A.2})$$

It is convenient to parametrize the special Kähler scalar fields with holomorphic symplectic sections of a projective bundle, $(X^\Lambda(z), F_\Lambda(z))^T$, $\Lambda = 0, 1, \dots, n_V$, satisfying

⁶Definitions and conventions used in the paper are explained in this appendix. For a complete discussion of $N = 2$ gauged supergravity we refer to the review[49]. For a more general analysis on gauged $N = 2$ vacua we refer to [20].

(bar indicates complex conjugation)

$$F_\Lambda \bar{X}^\Lambda - X^\Lambda \bar{F}_\Lambda = -ie^{-K} . \quad (\text{A.3})$$

The expression above defines the symplectic product as

$$\langle A_1, A_2 \rangle = A_1^T \Omega A_2 , \quad \Omega = \begin{pmatrix} 0 & I_{2n_V+2} \\ -I_{2n_V+2} & 0 \end{pmatrix} , \quad (\text{A.4})$$

on any $Sp(2n_v + 2)$ vector $A = (A_\Lambda, A^\Lambda)$. The normalized symplectic sections are then

$$\mathcal{V} = e^{K/2} (X^\Lambda(z), F_\Lambda(z))^T , \quad \langle \mathcal{V}, \bar{\mathcal{V}} \rangle = -i , \quad (\text{A.5})$$

they satisfy

$$\begin{aligned} D_i \mathcal{V} &= \partial_i \mathcal{V} + \frac{1}{2} \partial_i K \mathcal{V} , & D_{\bar{i}} \mathcal{V} &= \partial_{\bar{i}} \mathcal{V} - \frac{1}{2} \partial_{\bar{i}} K \mathcal{V} = 0 , \\ D_{\bar{i}} \bar{\mathcal{V}} &= \partial_{\bar{i}} \bar{\mathcal{V}} + \frac{1}{2} \partial_{\bar{i}} K \bar{\mathcal{V}} , & D_i \bar{\mathcal{V}} &= \partial_i \bar{\mathcal{V}} - \frac{1}{2} \partial_i K \bar{\mathcal{V}} = 0 . \end{aligned} \quad (\text{A.6})$$

By using the special geometry identity

$$D_{\bar{j}} D_i \mathcal{V} = g_{i\bar{j}} \mathcal{V} , \quad (\text{A.7})$$

one can derive the following relations used in Sec.4

$$\begin{aligned} \frac{2\partial_{\bar{j}} \partial_i |W|}{|W|} \Big|_{\partial_i |W|=0} &= g_{i\bar{j}} \Big|_{\partial_i |W|=0} , \\ \partial_{\bar{j}} \partial_i |W|^{p-2} \Big|_{\partial_i |W|=0} &= g_{i\bar{j}} (p-2) |W|^{p-2} \Big|_{\partial_i |W|=0} . \end{aligned} \quad (\text{A.8})$$

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